Acoustic Doppler Velocimeter Flow Measurement from an Autonomous Underwater Vehicle with Applications to Deep Ocean Convection

YANWU ZHANG*

MIT/Woods Hole Oceanographic Institution Joint Program in Oceanographic Engineering, and MIT Sea Grant Autonomous Underwater Vehicles Laboratory, Cambridge, Massachusetts

KNUSTREITLIEN+ AND JAMES G. BELLINGHAM#

MIT Sea Grant Autonomous Underwater Vehicles Laboratory, Cambridge, Massachusetts

ARTHUR B. BAGGEROER

Department of Ocean Engineering, and Department of Electrical Engineering and Computer Science, MIT, Cambridge, Massachusetts

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ABSTRACT

The authors present a new modality for direct measurement of ocean flow, achieved by combining the resolution and precision of an acoustic Doppler velocimeter with the mobility of an autonomous underwater vehicle. To obtain useful measurements, two practical integration issues must first be resolved: the optimal location for mounting the velocimeter probe to the vehicle, and alignment of the velocimeter with the vehicle’s attitude sensors. Next, it is shown how to extract earth-referenced flow velocities from the raw measurements. Vehicle hull’s influence on flow velocity measurement is removed by modeling the vehicle as a spheroid in potential flow. This approach was verified by a tow tank experiment in the David Taylor Model Basin. The authors then describe mission configuration and signal processing for a deployment in the Labrador Sea to study deep ocean convection during the winter of 1998. Analysis of the vertical flow velocity data not only shows that this form of measurement can detect ocean convection, but also gives detailed information about convection’s spatial structure.

1. Introduction

Autonomous underwater vehicles (AUVs) have inherent qualities that make them uniquely adapted to sampling fine-scale ocean phenomena (Bellingham 1997). Unencumbered by tethers and controlled by programmable mission specifications, AUVs possess a high degree of agility and can execute very precise surveys. Enhanced power sources are extending AUVs’ operation range to the order of 1000 km. Acoustic, radio, and satellite communications have made it possible to monitor and reconfigure AUV missions from afar. AUVs accept a variety of off-the-shelf payloads, as long as they have a reasonable power consumption and are small and light enough to be fitted. One such device is an acoustic Doppler velocimeter (ADV). Flow velocity is one of the most important physical quantities for ascertaining the ocean’s state and its circulation. An ADV (SonTek 1997) measures three-dimensional water velocity using sound waves.

As illustrated in the left-hand panel of Fig. 1, an ADV is an active sonar that transmits acoustic signals and then receives their echoes reflected by sound scatterers in the water, like plankton or other floating particles. The scatterers are usually passively advected by water motion (Gordon 1996; Plimpton et al. 1997). From the measured frequency shift between emissions and echoes, flow velocity is deduced based on the Doppler principle.

An ADV has one transmitter and three receivers, as shown in Fig. 1. An echo at each receiver has a frequency shift proportional to the component of relative velocity along a line bisecting the transmission and reflection paths. Hence, three-dimensional velocity can be calculated. Acoustic beams of the transmitter and the...
TABLE 1. Specifications for SonTek ADVOcean.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound frequency</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Output data rate</td>
<td>0.1 – 25 Hz</td>
</tr>
<tr>
<td>Velocity’s dynamic range (x and y)</td>
<td>0–0.5, 1.2, 2.0, 6.0, 7.2 m s⁻¹ programmable</td>
</tr>
<tr>
<td>Velocity’s dynamic range (z)</td>
<td>¼ of above</td>
</tr>
<tr>
<td>Measurement noise (at 25-Hz data rate)</td>
<td>1% of velocity range</td>
</tr>
<tr>
<td>Velocity resolution</td>
<td>10⁻⁴ m s⁻¹</td>
</tr>
<tr>
<td>Distance of sampling volume from transmitter</td>
<td>0.16 m</td>
</tr>
<tr>
<td>Sampling volume size</td>
<td>2 cm³</td>
</tr>
<tr>
<td>Depth rating</td>
<td>2000 m</td>
</tr>
<tr>
<td>Size</td>
<td>0.36 m × 0.18 m (diameter of stem: 0.05 m)</td>
</tr>
<tr>
<td>Weight</td>
<td>1.5 kg</td>
</tr>
</tbody>
</table>

Three receivers intersect at a small sampling volume (<2 cm³, deemed a “point”) located away from the instrument base (16-cm distance for SonTek model ADVOcean, as listed in Table 1). Three-dimensional flow velocity obtained at this distant focal point can thus be considered undisturbed by the probe. As shown in Fig. 1, the ADV’s local z axis is defined along the probe stem; the x axis is coplanar with one designated receiver arm; the y axis is accordingly defined by the right-hand rule. Table 1 gives the specifications (SonTek 1997) of the ADV device that we have installed in an Odyssey IIB AUV (Bellingham 1997).

It should be mentioned that an acoustic Doppler current profiler (ADCP) is another common flow velocity measurement device. It profiles velocity over a large depth range, in contrast to an ADV’s point measurement. However, an ADV’s spatial focus and low noise (less than a tenth of an ADCP’s measurement noise at the smallest bin size) makes it uniquely suitable for experiments that require high resolution and high precision. Published applications include flow measurement near river beds (Lane et al. 1998; Bouckaert and Davis 1998), and the seabed (Kawanis and Yokosi 1997). In Voulgaris and Trowbridge (1998), Reynolds stress measured by an ADV was found to be within 1% of that obtained by laser Doppler velocimetry.

In the above applications, ADVs monitor current velocity only at spatially fixed positions. Integration of an ADV into an AUV enables point flow measurement of high resolution and high precision from a moving platform. The paper presents theoretical and experimental work in building such a system. System integration is
introduced in section 2. The data processing algorithm is derived in section 3.

The Labrador Sea lies between northern Canada and Greenland. It is one of the few locations in the world where open ocean convection occurs (Marshall et al. 1998; Lilly et al. 1999). During the winter, the sea surface is subjected to intense heat flux to the atmosphere. The resulting buoyancy loss causes the surface water to sink to large depths, initiating ocean convection. In convection regimes, the water column overturns in numerous convective cells (also called plumes; Marshall and Schott 1999). Therefore, vertical flow velocity and its spatial periodicity are the key signatures of ocean convection.

Previous measurements by floats and moorings show that convection's vertical flow velocity is on the order of several centimeters per second (mainly depending on surface heat flux and latitude). For example, Fig. 20 of Marshall et al. (1998) shows rms vertical velocity of 2.3 cm s\(^{-1}\) measured by floats in the Labrador Sea in 1997.

During January–February 1998, researchers from the Massachusetts Institute of Technology (MIT), the Woods Hole Oceanographic Institution, and the University of Washington, made an expedition to the Labrador Sea to study ocean convection. This experiment provided a unique opportunity of testing and utilizing the AUV-borne ADV system. The challenge was to measure small vertical flows from a comparatively rapidly moving vehicle. The AUV-measured flow velocity detected convection’s occurrence, as demonstrated in section 4. Computation of AUV hull’s influence and a tow tank validation experiment are given in the appendixes.

2. Instrument integration

The mounting location and orientation of an ADV on an AUV must meet the following requirements: (i) avoid saturation of velocity measurement, (ii) avoid interference with other AUV instruments, (iii) keep the ADV probe out of harm’s way during AUV launch and recovery, (iv) minimize corrections of the vehicle hull’s influence so that the measurements are as direct as possible, (v) locate the sampling volume outside wakes as much of the time as possible, and (vi) exploit the lower (by a factor of 1/4) measurement noise along ADV’s \(z\) axis than the other two axes.

According to the above requirements, the following options are ruled out. (i) At AUV’s nose. The vehicle’s up to 2 m s\(^{-1}\) speed could saturate the ADV’s \(z\) velocity with the probe in the alongship direction. The ADV would also interfere with and be affected by the AUV’s docking latch and the ultrashort-baseline hydrophone array that are mounted at the nose. The velocity measurement would need considerable corrections because of the ADV’s closeness to the vehicle’s stagnation point. (ii) At AUV’s top or bottom. Either upward or downward flow velocity measurement would be in the wake, and the AUV’s launch gear or the storage cart would jeopardize the ADV probe. (iii) At the horizontal-plane flank of the vehicle. The ADV probe would interfere with the vehicle’s recovery hoop.

Our choice is to mount the ADV at the vehicle’s largest vertical cross section, with its probe pointing 45° from the vehicle’s horizontal central plane, as shown in the right-hand panel of Fig. 1. The ADV’s three receiver tips reach the brink of the hull but do not protrude beyond it. This satisfies all the requirements listed above. Inside the vehicle, the ADV probe is mounted with a horizontal plate and a 45° slanted bracket.

The vehicle’s heading and attitude sensor box is oriented in the alongship direction. For transformation from the ADV’s to the AUV’s reference frame (as will be formulated in section 3), the ADV’s mounting angles need to be accurate. During installation, we use a laser pointer to ensure alignment accuracy.

The ADV works at 24 V, provided by the AUV’s batteries. ADV data are read by the vehicle’s computer through a serial port and saved in the vehicle’s central...
data file. This guarantees time synchronization with other instrument data. The bandwidth of the AUV’s data bus limits the ADV’s data output rate to 2.5 Hz.

3. Algorithm of extracting earth-referenced flow velocity from AUV’s raw measurements

The AUV-borne ADV measures flow velocity relative to the moving vehicle. Hence, to obtain the earth-referenced flow velocity, that is, the true flow velocity, we must subtract the vehicle’s own velocity from the raw measurement. Another issue of concern is the vehicle hull’s influence on the measurement. As shown in Fig. 1, the ADV probe’s sampling volume is located about 13 cm from the vehicle’s hull surface (an Odyssey IIB AUV has a length of 2.2 m, and a diameter of 0.6 m at its largest vertical cross section where the ADV is mounted). This distance is small enough to necessitate removal of the hull’s influence on flow measurement. In this section, we present the algorithm of extracting the earth-referenced flow velocity.

Figure 2 shows the plan view of the spheroid that serves as our model of the AUV. We choose the spheroid to be of the same aspect ratio as the vehicle’s horizontal central plane. The vehicle’s motion relative to the water surrounding it imparts a disturbance at \( r_m \). The flow velocity that the ADV measures, \( u_m \), therefore has a difference from \( U - v_m \). According to the potential flow theory (Lamb 1932), the disturbance is a linear combination of the components of \( V - U \) and \( \Omega \). So \( u_m \) equals \( U - v_m \) plus a correction term:

\[
\mathbf{u}_m = \mathbf{U} - \mathbf{v}_m + [\mathbf{A}(\mathbf{V} - \mathbf{U}) + \mathbf{B}\mathbf{\Omega}]
\]

\[
= (\mathbf{A} - \mathbf{I})(\mathbf{V} - \mathbf{U}) + (\mathbf{B} + \mathbf{r}_m \times \mathbf{\Omega}), \tag{2}
\]

where Eq. (1) has been incorporated. Here \( \mathbf{A} \) and \( \mathbf{B} \) are two square matrices describing the AUV hull’s effect on flow velocity measurement: \( \mathbf{A} \) for translational motion and \( \mathbf{B} \) for rotational motion. Calculation of the two matrices is given in appendix A, and the results are

\[
\mathbf{A} = \begin{bmatrix}
-0.0678 & 0 & 0 \\
0 & 0.0563 & 0.4053 \\
0 & 0.4053 & 0.0563 
\end{bmatrix} \tag{3}
\]

\[
\mathbf{B} = \begin{bmatrix}
0 & 0.0687 & 0 \\
0 & 0 & 0.0687 \\
0 & 0 & 0 
\end{bmatrix} \tag{4}
\]

Thus, for instance, if the vehicle is moving forward with velocity \([1 0 0]^T\) relative to the ambient current, the ADV’s sampling volume will see a velocity of \([-1.068 0 0]^T\) due to the accelerated flow at the hull’s maximum diameter.

To validate the above theoretical modeling of vehicle hull’s influence on flow velocity measurement, we conducted a tow tank experiment in the David Taylor Model Basin, as detailed in appendix B. Experimental results agree well with theoretical predictions.

The earth-referenced flow velocity is extracted through the following steps.

1) Transform the velocity measurement from the ADV coordinate system to the AUV coordinate system (both systems shown in Fig. 1):

\[
\mathbf{u}_m = \mathbf{T}_{\text{ADV} \rightarrow \text{AUV}} \mathbf{u}_{\text{ADV}} \tag{5}
\]

where \( \mathbf{u}_{\text{ADV}} \) is the velocity vector originally measured in the ADV coordinate system.

Matrix

\[
\mathbf{T}_{\text{ADV} \rightarrow \text{AUV}} = \begin{bmatrix}
0 & -1 & 0 \\
-\sin(\alpha) & 0 & -\cos(\alpha) \\
\cos(\alpha) & 0 & -\sin(\alpha)
\end{bmatrix} \tag{6}
\]

transforms from the ADV coordinate system to the AUV coordinate system, where \( \alpha = 45^\circ \) is the mounting angle of the ADV probe’s stem relative to the vehicle’s horizontal central plane.

2) Compensate for the AUV hull’s influence and subtract the velocity induced by the vehicle’s rotation, by applying Eqs. (2) and (5). Then the relative flow velocity in the AUV coordinate system is obtained:
\[ \mathbf{U} - \mathbf{V} = (\mathbf{I} - \mathbf{A})^{-1} [\mathbf{u}_m - (\mathbf{B} + \mathbf{r}_m \times )\mathbf{\Omega}] \]

where the angular velocity (i.e., the vehicle’s yaw/pitch/roll rate) vector \( \mathbf{\Omega} \) is measured by the AUV’s KVH Digital Gyro Compass and Digital Gyro Inclinometer (KVH 1994).

3) Recover the relative flow velocity in the earth coordinate system using the vehicle’s heading, pitch, and roll measurements:

\[ \mathbf{U}_{\text{earth}} - \mathbf{V}_{\text{earth}} = (\mathbf{I} - \mathbf{A})^{-1} \times [\mathbf{T}_{\text{ADV-to-AUV}} \mathbf{u}_{\text{ADV}} - (\mathbf{B} + \mathbf{r}_m \times )\mathbf{\Omega}] \]  

where

\[ \mathbf{T}_{\text{ADV-to-earth}} = \mathbf{T}_{\text{h}} \mathbf{T}_{\text{p}} \mathbf{T}_{\text{r}} = \begin{bmatrix} \cos(\theta_h) & -\sin(\theta_h) & 0 \\ \sin(\theta_h) & \cos(\theta_h) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} \cos(\theta_p) & 0 & \sin(\theta_p) \\ 0 & 1 & 0 \\ -\sin(\theta_p) & 0 & \cos(\theta_p) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]

\[ \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) & 0 \\ \sin(\theta_r) & \cos(\theta_r) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta') \\ -\sin(\theta') \\ \cos(\theta') \end{bmatrix} \]

\[ w_{\text{down-earth}} = u_3 + u_{\text{down-AUV}} \]

\[ = u_3 + \frac{d}{dt} [\text{AUV depth } z(t)]. \]

To comply with the commonly adopted upward convention, the earth-referenced upward flow velocity is written as

\[ w_{\text{earth}} = -w_{\text{down-earth}} = \frac{d}{dt} [\text{AUV depth } z(t)]. \]

4. Labrador Sea convection experiment

a. Data processing results of AUV-measured vertical flow velocity

AUV mission B9804107 took place at 0446 – 0700 UTC 10 February 1998. The mission launch location was about 56°42′N, 52°46′W, where the (Autonomous Oceanographic Sampling Network (AOSN; Curtin et al. 1993) mooring was anchored. The AUV behaviors in this mission are illustrated in Fig. 3. The vehicle first spiraled down to 426-m depth, then it spiraled up to 250-m depth. At this depth plane, the vehicle made a “diamond” run, that is, closed a four-leg loop with 90° turns, each leg lasting for 720 s. After that, it spiraled up to 20-m depth, making an identical diamond run. At the end, the vehicle ascended to the sea surface. The vehicle’s speed in level legs was about 1 m s⁻¹.

On the vehicle, the ADV probe measured three-dimensional flow velocity, while the conductivity–temperature–depth (CTD) sensor suite measured other water properties. Those measurements are shown in Fig. 4.
where potential temperature and potential density are deduced from in-situ measurements.

We apply the algorithm in section 3 to process the ADV measurements. Vertical flow velocity is a key signature of ocean convection (Schott et al. 1993, 1996). For detecting convection, we only extract the earth-referenced vertical flow velocity $w_{\text{earth}}$ as formulated in Eq. (12). The data processing result for the 250-m depth is shown in Fig. 5. The vehicle’s own vertical velocity (middle panel) has been removed for producing $w_{\text{earth}}$ (top panel). The AUV’s roll (bottom panel) shows when the vehicle made 90° turns. At an AUV speed of about 1 m s$^{-1}$, 1 s of time on the abscissa can be approximately translated to 1 m of distance for interpreting convection cells’ spatial scale. The rigorous time–space relationship of an AUV measurement is governed by the mingled spectrum principle (Zhang 2000).

In Fig. 6, we overlap $w_{\text{earth}}$ of the 250- and 20-m depths. A shift of 3550 s is added to the time index for the 250-m depth, during which the vehicle completed the diamond loop at the 250-m depth and ascended to the 20-m depth plane (illustrated in Fig. 3). We note that for about 600 s in Fig. 6, consistency between the two signals is pronounced.

As will be analyzed in section 4b, a convection layer is believed to exist in the upper 350-m water during this AUV mission. According to the MIT ocean convection model (Jones and Marshall 1993; Klinger and Marshall 1995), over a depth difference of 230 m and a time difference of 3550 s, vertical velocities at the same horizontal position should show strong similarity. The reason is that convective cells are vertically aligned, and the field varies very slowly as convection approaches a stationary state. Compared with 250-m depth, the 20-m depth shallow water is complicated by additional atmospheric forcing. Nevertheless, similarity between depths still offers a means for checking measurements. The consistency of $w_{\text{earth}}$ between the two depths as shown in Fig. 6, although only lasting for a short duration, adds to our confidence in the data processing algorithm.

To remove bias in $w_{\text{earth}}$, we have corrected installation errors of the ADV probe and the heading and attitude sensor box. In AUV mission B9804107, the two errors
FIG. 5. (top) The earth-referenced vertical flow velocity $w_{\text{earth}}$ at the 250-m depth of AUV mission B9804107.

FIG. 6. Comparison of $w_{\text{earth}}$ of 250- and 20-m depths, with a time shift of 3550 s corresponding to AUV’s cruise time.
were 2° rotation of the ADV probe and 1° pitch-up of the heading and attitude sensor box. The pitch-up angle of the heading and attitude sensor box was determined using the criterion that the mean value of convective $w_{\text{earth}}$ during an AUV level run should vanish. The remaining noise of $w_{\text{earth}}$ comes from three sources of measurement noise: (i) ADV, (ii) KVH heading/pitch/roll and rate sensor, and (iii) AUV’s depth sensor. The source errors propagate into the final result $w_{\text{earth}}$ by following the four-step transformations in section 3. The noise sources and the result (all after 50-s smoothing) are summarized in Table 2. The standard deviation of noise in $w_{\text{earth}}$ is 0.7 cm s$^{-1}$.

It is noted that in the Labrador Sea Experiment the AUV-borne ADV experienced low correlation coefficient and low signal-to-noise ratio, although still within operational ranges. This was probably due to low density of floating particles in the water. As a consequence, ADV’s measurement noise was relatively high. Interference from an acoustic modem’s transmission also added to the ADV’s measurement noise during that experiment. Modem interference with the ADV was easy to remove, because it was periodic (every 28 s), and created clearly anomalous spikes in velocity measurement. Even in the presence of the 0.7 cm s$^{-1}$ total noise in $w_{\text{earth}}$, the AUV-based classifier successfully detects convection against internal waves, as will be discussed toward the end of this section.

**b. Detection of convection’s occurrence by $w_{\text{earth}}$**

During the 1998 Labrador Sea Experiment, meteorological data were recorded by an Improved Meteorological (IMET) system (Hosom et al. 1995) on board R/V Knorr. Prof. Peter Guest of the Naval Postgraduate School calculated the ocean surface heat flux based on the IMET measurements. During AUV mission B9804107, the heat flux was about 300 W m$^{-2}$. Let us make comparisons with previous open ocean convection experiments. In the Greenland Sea Experiment (Schott et al. 1993) during the winter of 1988/89, the heat flux fluctuated between 100 and 500 W m$^{-2}$, with an average value of about 250 W m$^{-2}$. Ocean convection was observed during that experiment, using moored ADCPs. In an earlier Labrador Sea experiment (Lilly et al. 1999) during the winter of 1994/95, the average heat flux was about 300 W m$^{-2}$. Using a moored ADCP and Profiling Autonomous Lagrangian Circulation Explorer (PALACE) floats, ocean convection was observed. The sea surface heat flux value in our Labrador Sea Experiment is close to that of the two previous experiments. We therefore have reason to expect ocean convection’s occurring.

Besides surface heat loss, a vertically mixed water column is another key condition for ocean convection. Across the Labrador Sea Basin (about 600 km), a mixed water layer of depth 270–500 m was observed by a series of CTD casts from the ship deck (data provided by Prof. Eric D’Asaro, personal communication 2000). During two different AUV missions, mixed water layers were also clearly recorded by CTD sensors on the vehicle. During AUV mission B9804107, the mixed layer was down to 350 m, as shown in Fig. 7. The 250-m depth plane in AUV mission B9804107 is within this mixed layer.

From the $w_{\text{earth}}$ panel in Fig. 5, convection cells’ spatial periodicity of about 150 m can be observed (the AUV speed was about 1 m s$^{-1}$). Using the MIT convection model (Jones and Marshall 1993; Klinger and Marshall 1995), we have designed an AUV-based classifier (Zhang 2000) to distinguish ocean processes by their spectral features. Based on $w_{\text{earth}}$’s spectrum, which corresponds to convective cells’ periodicity “seen” by a cruising AUV, the classifier declares that the 250-m depth vertical velocity data is convective. This result is in agreement with meteorological and hydrographic expectations.

During the same experiment, Prof. Eric D’Asaro of the University of Washington deployed seven Lagrangian floats to study convection [float design can be found in D’Asaro et al. (1996)]. The floats’ records confirm not only the existence of mixed layers, but also the

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**Table 2. Measurement/estimation noise.**

<table>
<thead>
<tr>
<th>Measurement/estimation</th>
<th>Sensor</th>
<th>Rms noise after 50-s smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow velocity</td>
<td>ADV/Ocean</td>
<td>0.45 cm s$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>5 MHz</td>
<td></td>
</tr>
<tr>
<td>Heading/attitude rate</td>
<td>KVH</td>
<td>1.4 $\times$ 10$^{-3}$ rad s$^{-1}$</td>
</tr>
<tr>
<td>AUV vertical velocity</td>
<td>Paroscientific 8B-4000</td>
<td>0.32 cm s$^{-1}$</td>
</tr>
</tbody>
</table>

* For AUV mission B9804107 in the 1998 Labrador Sea Experiment.

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**Fig. 7. Profiles of potential temperature, salinity, and potential density during AUV mission B9804107.**
occurrence of convection. Furthermore, the rms vertical flow velocity is found to be 2–3 cm s⁻¹ based on the float data (calculated by Prof. Eric D’Asaro). In the 250-m depth AUV data analyzed above, the counterpart is 2 cm s⁻¹. Measurements by those two independent platforms are consistent.

5. Conclusions

We have enabled an AUV to make flow velocity measurement using an acoustic Doppler velocimeter (ADV). The usefulness of an ADV lies in its ability to acquire accurate measurement at a focal point. Incorporation of an ADV in an AUV provides a mobile measurement mode, which enhances coverage and responsiveness in ocean surveys. For classifying ocean processes, flow velocity is a key feature. Acquiring this measurement from a vehicle is crucial to realizing an AUV-based classifier (Zhang 2000).

The AUV-borne ADV measures flow velocity relative to the moving vehicle. We established an algorithm to extract the earth-referenced flow velocity from the raw measurement. AUV’s own velocity and its hull’s influence on flow measurement are removed. A tow tank validation experiment was carried out in the David Taylor Model Basin. In January/February 1998, an AUV acquired flow velocity in the Labrador Sea. Using a spectral classifier, convection is detected based on the acquired flow velocity in the Labrador Sea. Using AUV’s own velocity and its hull’s influence on flow measurement are removed. A tow tank measurement. AUV’s own velocity and its hull’s influence on flow measurement are removed. A tow tank validation experiment was carried out in the David Taylor Model Basin. In January/February 1998, an AUV acquired flow velocity in the Labrador Sea. Using a spectral classifier, convection is detected based on the acquired flow velocity in the Labrador Sea. Using a spectral classifier, convection is detected based on the acquired flow velocity in the Labrador Sea.

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The natural coordinate system for the spheroid is the spheroidal coordinates, μ, ξ, ω, related to the Cartesian coordinates through

\[ x = k\mu\xi \]
\[ y = k\sqrt{1 - \mu^2\xi^2 - 1} \cos\omega \]
\[ z = k\sqrt{1 - \mu^2\xi^2 - 1} \sin\omega \]

where \( x = \pm k \) are the foci of the spheroidal coordinates. The spheroidal body is defined by its major and minor axes, \( a \) (along the \( x \) axis) and \( c \), its surface given by either

\[ \frac{x^2}{a^2} + \frac{y^2}{c^2} + \frac{z^2}{c^2} = 1 \]

or

\[ \xi = \xi_0. \]

We have

\[ k = a\sqrt{1 - c^2/a^2}, \quad \xi_0 = a/k. \]

The velocity potentials found by Lamb (1932) are most compactly written by mixing the coordinates with \( \xi(x, y, z) \) as an intermediate variable. In order to extract the velocities in the direction of the Cartesian axes, we must evaluate \( \nabla\phi \) where \( \phi \) is a function of \( x, y, z \) only. Hildebrand (1976) shows how to apply the chain rule for such a case:

\[ \frac{\partial\phi}{\partial x}\bigg|_{y,z} = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial x}, \quad \frac{\partial\phi}{\partial y}\bigg|_{x,z} = \frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial y}, \quad \frac{\partial\phi}{\partial z}\bigg|_{x,y} = \frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial z}. \]
approach, differentiating both sides of Eq. (A7): the derivative of $z$ as if $z$ were independent.

\[ \tan \omega = \frac{z}{y} \quad (A5) \]

Direct differentiation of $\zeta$ with respect to the Cartesian coordinates is tedious. Instead we use an implicit approach, differentiating both sides of Eq. (A7):

\[ 2k^2 \zeta d\zeta = rdr + \frac{(r^2 + k^2)rdr - 2k^2 xd\zeta}{\sqrt{(r^2 + k^2)^2 - 4k^2x^2}} \quad (A8) \]

where

\[ rdr = xd\zeta + yd\zeta + zd\zeta. \]

Combination of Eqs. (A6) and (A7) simplifies Eq. (A8):

\[ 2k^2 \zeta d\zeta = rdr + \frac{(\zeta^2 + \mu^2)rdr - 2xd\zeta}{\zeta^2 - \mu^2} \quad (A9) \]

Since by definition

\[ d\zeta = \frac{\partial \zeta}{\partial x} dx + \frac{\partial \zeta}{\partial y} dy + \frac{\partial \zeta}{\partial z} dz, \]

we can conclude

\[ \frac{\partial \zeta}{\partial x} = \frac{\zeta - 1/\zeta}{\zeta^2 - \mu^2 k^2} \quad (A10) \]

\[ \frac{\partial \zeta}{\partial y} = \frac{\zeta}{\zeta^2 - \mu^2 k^2} \quad (A11) \]

\[ \frac{\partial \zeta}{\partial z} = \frac{\zeta}{\zeta^2 - \mu^2 k^2}; \quad (A12) \]

Table A1 shows, for each degree of freedom, $j$, the expressions for the potential $\phi$ and its amplitude $A^3$.

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**Table A1. Potentials and velocities for flow around a spheroid.**

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\phi$</th>
<th>$A$</th>
<th>$u_x$</th>
<th>$u_y$</th>
<th>$u_z$</th>
<th>$\frac{\partial \phi}{\partial x}$</th>
<th>$\frac{\partial \phi}{\partial y}$</th>
<th>$\frac{\partial \phi}{\partial z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{2} \ln \frac{\zeta + 1}{\zeta - 1} - \frac{1}{\zeta}$</td>
<td>$\frac{1}{2} \ln \frac{\zeta_0 + 1}{\zeta_0 - 1} - \frac{\zeta_0}{\zeta_0 - 1}$</td>
<td>$\frac{\partial \phi}{\partial x}$</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
<td>$\frac{\partial \phi}{\partial z}$</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2} \ln \frac{\zeta + 1}{\zeta - 1} - \frac{\zeta}{\zeta^2 - 1}$</td>
<td>$\frac{1}{2} \ln \frac{\zeta_0 + 1}{\zeta_0 - 1} - \frac{\zeta_0}{\zeta_0 - 1}$</td>
<td>$\frac{\partial \phi}{\partial x}$</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
<td>$\frac{\partial \phi}{\partial z}$</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{2} \ln \frac{\zeta + 1}{\zeta - 1} - \frac{\zeta}{\zeta^2 - 1}$</td>
<td>$\frac{1}{2} \ln \frac{\zeta_0 + 1}{\zeta_0 - 1} - \frac{\zeta_0}{\zeta_0 - 1}$</td>
<td>$\frac{\partial \phi}{\partial x}$</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
<td>$\frac{\partial \phi}{\partial z}$</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{3}{2} \ln \frac{\zeta + 1}{\zeta - 1} - \frac{3\zeta - 2}{\zeta^2 - 1}$</td>
<td>$\frac{3}{2} \ln \frac{\zeta_0 + 1}{\zeta_0 - 1} - \frac{6\zeta_0^2 - 7\zeta_0}{\zeta_0 - 1}$</td>
<td>$\frac{\partial \phi}{\partial x}$</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
<td>$\frac{\partial \phi}{\partial z}$</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{3}{2} \ln \frac{\zeta + 1}{\zeta - 1} - \frac{3\zeta - 2}{\zeta^2 - 1}$</td>
<td>$\frac{3}{2} \ln \frac{\zeta_0 + 1}{\zeta_0 - 1} - \frac{6\zeta_0^2 - 7\zeta_0}{\zeta_0 - 1}$</td>
<td>$\frac{\partial \phi}{\partial x}$</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
<td>$\frac{\partial \phi}{\partial z}$</td>
<td>$\frac{\partial \phi}{\partial x}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial y}$ / $\zeta$</td>
<td>$\frac{\partial \phi}{\partial z}$ / $\zeta$</td>
</tr>
</tbody>
</table>

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$A^3$ Compared with Lamb’s formulation, we use the opposite sign for $\phi$, restrict motions to unit velocity, and absorb a factor $k$ or $k^2$ into $A$. Our amplitudes therefore differ from Lamb’s by a factor $-1/k^2$ for the translational degrees of freedom, and $-1/k^2$ for the rotational degrees of freedom.
adapted from Lamb (1932). Listed next are the non-
zero terms in the chain rule (A4) for the velocities. The
seventh to ninth columns show the derivatives of \( \phi \) with
respect to \((x, y, z)\) as needed in the chain rule. Because
each of these coordinates only appears linearly in the
potentials, the derivatives are compactly written as \( \phi/x, \)
etc. Finally, the derivative of \( \phi \) with respect to \( \xi \) is shown.
With the coordinate derivatives from Eqs. (A10)–(A12),
any flow velocity component \( (u_1, u_2, u_3) \) due to unit
motion in any degree of freedom can then be computed.\(^{44}\)

In our case, \( a/c = 1/0.293 \), and \( (x, y, z) = (0, 0.304a, 0.304a) \), leading to

\[
\begin{align*}
  u_{11} &= -0.0678 & u_{12} &= 0.0000 & u_{13} &= 0.0000 \\
  u_{21} &= 0.0000 & u_{22} &= 0.0563 & u_{23} &= 0.4053 \\
  u_{31} &= 0.0000 & u_{32} &= 0.4053 & u_{33} &= 0.0563 \\
  u_{44} &= 0.0000 & u_{45} &= 0.0687a & u_{46} &= -0.0687a \\
  u_{54} &= 0.0000 & u_{55} &= 0.0000 & u_{56} &= 0.0000 \\
  u_{64} &= 0.0000 & u_{65} &= 0.0000 & u_{66} &= 0.0000.
\end{align*}
\]

These are the components of the disturbance matrices
\( A \) [Eq. (3)] and \( B \) [Eq. (4)], respectively.

\(^{44}\) MATLAB routines that evaluate \( \phi \) and \( u_\xi \), as well
as examples of their use, can be downloaded from ftp://ftp.mathworks.com/pub/
contrib/v5/physics/pfs/.
APPENDIX B

Calibration Experiment at the David Taylor Model Basin

To ascertain the AUV hull’s influence on the flow velocity measurement, we carried out a calibration experiment in the David Taylor Model Basin before the Labrador Sea Experiment. The tow tank has a large cross section to minimize influence from the boundaries, and the carriage has a precise speed control. In the tank, a complete AUV hull equipped inside with an ADV was towed by the carriage at different attack angles under different speeds.

a. Experiment design

Let us present the experimental setup in relation to Eq. (2) and Fig. 2. In consideration of the available facilities, we did not attempt to generate the vehicle’s rotational motion, so \( \Omega = 0 \) in Eq. (2). The tank water is still, so \( \mathbf{U} = 0 \). Then Eq. (2) is simplified to

\[
\mathbf{u}_m = (\mathbf{A} - \mathbf{I}) \mathbf{V}.
\]  

(B1)

Matrix \( \mathbf{A} \), as given in Eq. (3), is computed by the potential flow theory described in appendix A. This matrix depicts the AUV hull’s influence on the flow velocity measurement, which is induced by the vehicle’s translational motion. At a series of AUV speeds, ADV-measured flow velocities are to be compared with theoretical predictions computed by Eq. (B1) using matrix \( \mathbf{A} \) in Eq. (3). To test out various flow orientations relative to the AUV, we need to enable different combinations of the vehicle’s yaw and pitch angles.

The experimental structure is illustrated in Fig. B1. The structure is composed of three parts: a rotating bracket, a wedge, and a hull platform. The upper rectangular plate of the rotating bracket attaches the whole load to the tow tank carriage. Its lower circular plate connects to the wedge via four bolts. This circular plate has multiple bolt holes to permit yaw angles of 0°, 5°, 10°, 15°, 30°, and 45°. The wedge is for realizing the AUV’s pitch/roll angles of 5°, 10°, 15°, and 30°. The hull platform’s upper circular plate connects to the wedge, and its lower rectangular plate attaches to the AUV’s inner fairing. Its 45° slanted clamp (not visible in Fig. B1 from that perspective) holds the ADV probe. Two installation errors were found by two calibration runs (zero yaw and zero pitch with the AUV hull on, and zero yaw and zero pitch without the AUV hull): (i) 2.7° rotation of the ADV probe in the clamp, and (ii)
2.2° misalignment between the hull platform’s centerline and that of the AUV.

It should be noted that in this calibration experiment, the ADV probe pointed 45° upward on the vehicle’s port side, as shown in Fig. B1. In the Labrador Sea Experiment, the ADV probe pointed 45° downward on the vehicle’s starboard side, as shown in Fig. 1. The purpose of the change is to facilitate field recovery of the AUV at the end of missions, which requires contacts at the upper half of the vehicle. This difference is trivial, since it is equivalent to rotating the vehicle for 180° about its alongship axis. It can be shown that this diagonal move results (with the AUV’s latch on) and the theoretical predictions in \([\text{Equation (6)}]\). In the 1998 Labrador Sea Experiment, the ADV had a V-shaped latch at its nose for docking to an underwater station. We tested this double unfavorable orientation to determine whether wake induced by the latch had impact on flow measurement. In AUV cruises in the 1998 Labrador Sea Experiment, attack angle due to current flow was no more severe than the above setting. Tow tank experiment results in the fourth panel of Fig. B2 maintain agreement with theoretical predictions at all three speeds, not affected by the latch’s wake at the above orientation.

Relative errors between the experimental results and the theoretical predictions (corresponding to Fig. B2) are shown in Table B2, defined as

\[
\text{relative error} = \frac{|V_{\text{experiment}} - V_{\text{theory}}|}{|V_{\text{experiment}}|}\;
\]

where \(| \cdot |\) denotes the Euclidean norm. Relative errors at different carriage speed (1, 2, 3 knots) are averaged to give the tabulated values. Based on the good agreement between experimental results and theoretical predictions, the algorithmic step of removing AUV hull’s influence on flow measurement is validated.

### Table B1. Tested combinations of yaw and pitch angles (°).

<table>
<thead>
<tr>
<th>Pitch</th>
<th>−15</th>
<th>−5</th>
<th>0</th>
<th>5</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>−15°</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>*</td>
</tr>
<tr>
<td>−5°</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>0°</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5°</td>
<td>X*</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

X ADV Mounted in hull, latch attached.
* ADV only.
** ADV Mounted in hull, no latch.

### Table B2. Relative errors between experimental results and theoretical predictions.

<table>
<thead>
<tr>
<th>Pitch</th>
<th>−15°</th>
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<th>0°</th>
<th>5°</th>
<th>15°</th>
</tr>
</thead>
<tbody>
<tr>
<td>−15°</td>
<td>4%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>−5°</td>
<td>4%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>0°</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>5°</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
</tbody>
</table>

* Only at 1-knot carriage speed.

REFERENCES


